B. ANALYSIS OF PERFORMANCE

SECTION A

Question 1 **[10 × 3]**

(i) If (A – 2 I) (A – 3 I) = 0, where A = $\begin{pmatrix} 4 & 2 \\ -1 & x \end{pmatrix}$. and I = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find the value of x.

(ii) Find the value(s) of k so that the line $2x + y + k = 0$ may touch the hyperbola $3x^2 - y^2 = 3$.

(iii) Prove that: $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\sin^{-1}\frac{4}{5}$

(iv) Using L'Hospital's Rule, evaluate:

$$\lim_{x\to 0}\left(\frac{e^x - e^{-x} - 2x}{x - \sin x}\right)$$

(v) Evaluate: $\int \frac{1}{x + \sqrt{x}}\,dx$

(vi) Evaluate: $\int_0^1 \log\left(\frac{1}{x} - 1\right)dx$

(vii) Two regression lines are represented by $4x + 10y = 9$ and $6x + 3y = 4$.
Find the line of regression of y on x.

(viii) If 1, w and w^2 are the cube roots of unity, evaluate $(1 - w^4 + w^8)(1 - w^8 + w^{16})$

(ix) Solve the differential equation:

$$log\left(\frac{dy}{dx}\right) = 2x - 3y$$

(x) If two balls are drawn from a bag containing three red balls and four blue balls, find the probability that:

(a) They are of the same colour.

(b) They are of different colours.

Comments of Examiners

(i) Errors were made by candidates while multiplying matrices because of carelessness. Some could not equate the Left Hand Matrix with Null Matrix.

(ii) Mistakes were made while expressing the given hyperbola in the standard form (i.e., $\frac{x^2}{a_2} - \frac{y2}{b2} = 1$) and values of a^2, b^2 are incorrect. Some candidates used the wrong formula for condition of tangency of a line with hyperbola.

(iii) After obtaining tan-1 $\left(\frac{1}{2}\right)$ on the Left Hand Side many could not proceed further.

(iv) Some candidates used U/V rule instead of L' Hospital's Rule, as required. Derivative of e^{-x} was wrongly taken as e^{-x} instead of $-e^{-x}$.

(v) Errors were made by some candidates while using inappropriate substitutions. Some used rationalization of the denominator process and then tried to decompose in to partial Fractions without success.

(vi) Use of the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, was not made by many. Some used the method of integration by parts but this involved complicated steps which many could not handle.

(vii) Many candidates solved the equations unnecessarily and tried to identify b_{yx} arbitrarily. The condition for the two equations to represent regression lines and the tests for identifying them were not used by some.

(viii) Some candidates expanded directly but failed to simplify w^3 and higher degrees into simplest form before proceeding. Also use of $1+w+w^2= 0$ was not made by some.

(ix) Several candidates substituted √=2x-3y and thus made things more complicated. Some could not get $\frac{dy}{dx} = e^{2x-3y}$ from the given, log $\left(\frac{dy}{dx}\right)$ = 2x-3y. Separation of variables hence or otherwise was incorrect.

Suggestions for teachers

– Basic operations with Matrices need to be explained. Difference between Matrix and Determinant should be made clear by discussing separately with sufficient practice. The concept of equality of matrices and taking of only those values of the unknown variable that satisfy all conditions must be clear.

– If hyperbola is $\frac{x^2}{1} - \frac{y^2}{3} = 1$, then $a^2=1$, $b^2=3$

Condition for the line y = mx + c, to be a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, is, C = $\pm\sqrt{a^2m^2 - b^2}$.

These basics have to be taught clearly.

- Conversion of inverse circular functions (one to another) needs to be explained clearly to students. Plenty of practice is required to understand the applications of the laws, especially double angle laws.

- Applications of L'Hospital's Rule for calculating Limits of Indeterminate Forms $\left(\frac{0}{0} or \frac{\infty}{\infty}\right)$ should be taught well. In the rule the numerator and denominator need to differentiated separately till $\frac{0}{0}$ is removed.

- Teach properties of definite integrals well and see that the students learn to apply them appropriately. Properties when correctly used reduce cumbersome calculations into simple ones.

(x) Addition property was not used by some candidates. The probability of drawing 2 balls of the same colours implies the case of both being red and both being blue. Some failed to take these cases.

Some took the probability of different colours as $\frac{3}{7}\times\frac{4}{7}$ instead of $\left(\frac{{}^3c_1 \cdot {}^4c_1}{{}^7c_2}\right)$ or $\left(\frac{3}{7},\frac{4}{6}+\frac{4}{7},\frac{3}{6}\right)$.

MARKING SCHEME

Question 1.

(i) A = $\begin{pmatrix} 4 & 2 \\ -1 & x \end{pmatrix}$, I = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, A- 2I = $\begin{pmatrix} 2 & 2 \\ -1 & x-2 \end{pmatrix}$ A – 3I = $\begin{pmatrix} 1 & 2 \\ -1 & x-3 \end{pmatrix}$

$$(A-2I)\cdot(A-3I)=\begin{pmatrix} 2 & 2 \\ -1 & x-2 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & x-3 \end{pmatrix}=\begin{pmatrix} 0 & 2x-2 \\ -x+1 & x^2-5x+4 \end{pmatrix}$$

(A – 2I) (A– 3I) = 0 , hence 2x – 2 = 0 $\therefore x=1$

(ii) $3x^2 - y^2 = 3$

$$\frac{x^2}{1}-\frac{y^2}{3}=1$$

$a^2 = 1$, $b^2 = 3$

y = –2x – k

m = –2, c = –k

Condition for tangency:

$c^2 = a^2 m^2 - b^2$

$\Rightarrow k^2 = 4-3=1$

$k=\pm 1$

(iii) LHS = $\tan^{-1}\frac{1}{4}+\tan^{-1}\frac{2}{9}=\tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\cdot\frac{2}{9}}\right)$

$$=\tan^{-1}\frac{1}{2}=\frac{1}{2}\left(2\tan^{-1}\frac{1}{2}\right)$$

$$=\frac{1}{2}\cdot\sin^{-1}\frac{2\left(\frac{1}{2}\right)}{1+\left(\frac{1}{2}\right)^2}=\frac{1}{2}\sin^{-1}\frac{4}{5}$$

= RHS

(iv) $\lim_{x\to 0}\frac{e^x - e^{-x} - 2x}{x - \sin x}\left(\frac{0}{0}\right) = \lim_{x\to 0}\frac{e^x + e^{-x} - 2}{1 - \cos x}\left(\frac{0}{0}\right)$

$= \lim_{x\to 0}\frac{e^x - e^{-x}}{\sin x}\left(\frac{0}{0}\right) = \lim_{x\to 0}\cdot\frac{e^x + e^{-x}}{\cos x} = 2$

(v) $\int \frac{1}{x+\sqrt{x}}dx$

$= \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)}dx$ Put $t = \sqrt{x}+1$

dt = $\frac{1}{2\sqrt{x}}dx$

$= \int \frac{2dt}{t}$

$= 2\log|t| + c$

$= 2\log\left|\sqrt{x}+1\right| + c$

(vi) $I = \int_0^1 \log\left(\frac{1-x}{x}\right)dx$

$= \int_0^1 \log\frac{x}{1-x}dx$

$2I = \int_0^1 \left[\log\left(\frac{1-x}{x}\right) + \log\left(\frac{x}{1-x}\right)\right]dx$

$2I = \int_0^1 \log(1)dx$

$= 0$

$\therefore I = 0$

(vii) Let the line of regression of y on x be

4x + 10y = 9

$\therefore y = \frac{-4}{10}x + \frac{9}{10}$

$\therefore b_{yx} = \frac{-4}{10}$

The line of regression of x on y be

6x + 3y = 4

x = $-\frac{1}{2}y + \frac{2}{3}$

$\therefore b_{xy} = -1/2$

$\therefore r^2 = \frac{-4}{10} \times \frac{-1}{2} = \frac{1}{5} < 1$

$\therefore$ the line of regression of y on x is 4x + 10y = 9

(viii) $(1 - w^4 + w^8)(1 - w^8 + w^{16})$

$= (1 - w + w^2) \cdot (1 - w^2 + w)$

$= (-2w)(-2w^2)$

$= 4w^3 = 4$

(ix) $\log\left(\frac{dy}{dx}\right) = 2x - 3y$

$\frac{dy}{dx} = e^{2x-3y}$

$\therefore \frac{dy}{dx} = e^{2x} \times e^{-3y}$

$\Rightarrow \int e^{3y} dy = \int e^{2x} dx$

$\frac{e^{3y}}{3} = \frac{e^{2x}}{2} + c \quad \therefore 2e^{3y} - 3e^{2x} = k$

(x) $P(both\,red) = \frac{3C_2}{7C_2},\; P(both\,blue) = \frac{4C_2}{7C_2}$

(a) $P(both\,same\,colour) = \frac{3C_2}{7C_2} + \frac{4C_2}{7C_2} = \frac{3+6}{21} = \frac{3}{7}$

(b) $P(both\,different\,colour) = \frac{3C_1 \times 4C_1}{7C_2} = \frac{12}{21} = \frac{4}{7}$

Question 2

(a) Using properties of determinants, prove that: **[5]**

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$$

(b) Find A^{-1}, where $A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{bmatrix}$ **[5]**

Hence, solve the following system of linear equations:

$4x + 2y + 3z = 2$

$x + y + z = 1$

$3x + y - 2z = 5$

Comments of Examiners

(a) Errors were made by many candidates in use of Determinant properties (e.g. C_1 - C_2 and then C_2 - C_3 cannot be followed up by C_3 - C_1 with original elements of C_1).
Common factors in a line were not extracted by some.
Several candidates expanded directly. At times, correct cofactors are not used.

(b) Errors were made by several candidates while calculating the cofactors of the elements of 'A' and hence or otherwise the value of determinant of 'A'.
For finding the unknown matrix X, some candidates used post-multiplication with inverse of A. A few candidates solved using Cramer's Rule.

Suggestions for teachers

- Determinant expansion needs to be explained well. Students must be shown how to use the properties correctly. Use of properties and extraction of common factors when they appear, help in simplifying the determinant to make the final expansion easy.
- Instil the basics with regards to cofactors of elements, obtaining adjoint and inverse and meaning of pre– and post– multiplication of matrices; e.g. if AX=B then $X=A^{-1}B$ and not BA as multiplication of matrices is not commutative.

MARKING SCHEME

Question 2.

(a) $R_3 \to R_3 + R_1$

$$\Delta = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix} = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x+y+z & x+y+z & x+y+z \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} x-y & y-z & z \\ x^2-y^2 & y^2-z^2 & z^2 \\ 0 & 0 & 1 \end{vmatrix}$$ $c_1 \to c_1 - c_2$, $c_2 \to c_2 - c_3$

$$= (x+y+z)(x-y)(y-z)\begin{vmatrix} 1 & 1 & z \\ x+y & y+z & z^2 \\ 0 & 0 & 1 \end{vmatrix}$$

= (x+y+z)(x-y)(y-z)(·z-x)

(b)
$$|A| = \begin{vmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{vmatrix}$$

$$= 4(-2-1) - 2(-2-3) + 3(1-3)$$

$$= -12 + 10 - 6 = -8$$

co factors of row 1 : –3, 5, –2

co factors of row 2 : 7, –17, 2

co factors of row 3 : –1, –1 , 2

$$\text{Adj } A = \begin{bmatrix} -3 & 5 & -2 \\ 7 & -17 & 2 \\ -1 & -1 & 2 \end{bmatrix}^1$$

$$= \begin{bmatrix} -3 & 7 & -1 \\ 5 & -17 & -1 \\ -2 & 2 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{Adj\ A}{|A|} \qquad = -\frac{1}{8}\begin{bmatrix} -3 & 7 & -1 \\ +5 & -17 & -1 \\ -2 & 2 & 2 \end{bmatrix}$$

For the given system of equations

$$AX = B, \quad \text{where} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad and \quad B = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$$

$$\therefore X = A^{-1} B$$

$$X = \frac{-1}{8}\begin{bmatrix} -3 & 7 & -1 \\ 5 & -17 & -1 \\ -2 & 2 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{8}\begin{bmatrix} -4 \\ -12 \\ 8 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 3/2 \\ -1 \end{bmatrix}$$

$$\therefore x = \frac{1}{2}, y = \frac{3}{2}, z = -1$$

Question 3

(a) Solve for x: $\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$ **[5]**

(b) Construct a circuit diagram for the following Boolean Function: **[5]**

(BC+A)(A′B′+C′) + A′B′C′

Using laws of Boolean Algebra, simplify the function and draw the simplified circuit.

Comments of Examiners

(a) Generally, the formula for $\sin^{-1}x + \sin^{-1}y$ was correctly applied on the LHS. Errors were made by candidates while squaring, simplifying and solving higher degree algebraic equations. Some candidates converted all terms into $\tan^{-1}$ form but could not handle the resulting bulky fractions.

(b) Errors were made by some candidates in drawing the circuit. Boolean Algebra laws were generally well applied but some candidates erred using the distributive property and so failed to simplify (A+A'B') as (A+B').

Suggestions for teachers

– ICF laws need to be taught thoroughly for full understanding by the students. The applications need to be illustrated with copious examples. Inter conversion of functions is to be done if it helps the simplification process.

– Laws of Boolean Algebra need to be properly understood by all concerned. Different circuits for union and intersection need to be understood. Distributive property of 'union' over 'intersection' is unique to Boolean Algebra (sets) as, a+b.c = (a+b). (a+c) is not true in any other mathematical systems.

MARKING SCHEME

Question 3.

(a) $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$

$$\Rightarrow \sin^{-1}x + \sin^{-1}(1-x) = \frac{\text{®}}{2} - \sin^{-1}x$$

$$\sin^{-1}(1-x) = \frac{\text{®}}{2} - 2\sin^{-1}x$$

Let $\sin^{-1}x = y \quad \therefore x = \sin y$

$$\therefore \sin^{-1}(1-x) = \frac{\text{®}}{2} - 2y$$

$$1-x = \sin\left(\frac{\text{®}}{2} - 2y\right)$$

$$1-x = \cos(2y)$$

$$1-x = 1-2\sin^2 y$$

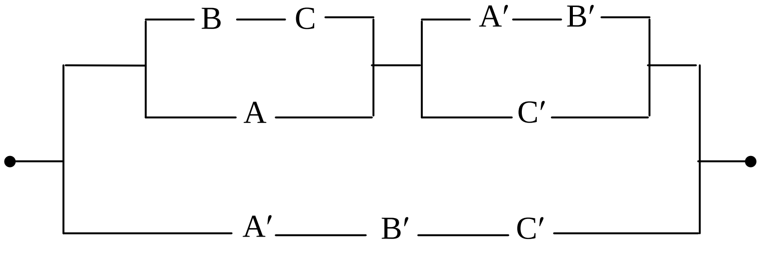
$$1-x = 1-2x^2$$

$$2x^2 - x = 0$$

$$x(2x-1) = 0$$

$$x=0,\ \frac{1}{2}$$

(b)



(BC + A)·(A′B′+C′) + A′B′C′

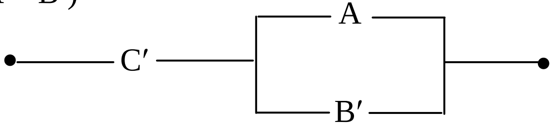
= BCA′B′+ BCC′ + AA′B + AC′ + A′B′C′

= 0 + 0 + 0 + AC′ + A′B′C′

= C′(A+A′B′)

= C′(A+A′) (A + B′)

= C′ (A + B′)



Question 4

(a) Verify Lagrange's Mean Value Theorem for the function f(x) = $\sqrt{x^2 - x}$ in the interval [1, 4]. **[5]**

(b) From the following information, find the equation of the Hyperbola and the equation of its Transverse Axis: **[5]**

Focus: (–2, 1), Directrix: $2x - 3y + 1 = 0$, e = $2/\sqrt{3}$

Comments of Examiners

(a) While defining the criteria for Lagrange's Mean Value Theorem, many candidates were unable to correctly attach the "closed interval" and "open interval" option with the relevant criterion. Some also differentiated $\sqrt{x^2 - x}$ incorrectly. That the value of 'C' must belong to the "open interval" was not found or stated by some.

(b) The fundamental relation PS = e.PM was reversed by some into PM = e.PS. Several candidates failed to use the value of 'e'. Errors also occurred while squaring and simplifying. Some candidates forgot to find the equation of the transverse axis.

Suggestions for teachers

- The different criteria for Rolle's and Lagrange's Mean Value theorems need to be understood and differentiated. Difference between 'closed' and 'open' intervals requires explanation. For Lagrange's Mean Value theorem $f'(c) = \frac{f(b) - f(a)}{b - a}$, and C has to be found in the open interval and so stated.
- Conics and their equations need to be thoroughly explained separately as well as jointly. Something as basic as axis and directrix of a conic cannot be forgotten, mistaken or ignored. Fundamental algebraic operations such as squaring of binomials or trinomials, simplification, solution etc. require plenty of practice.

MARKING SCHEME

Question 4.

(a) Given:

$$f(x) = \sqrt{x^2 - x}, x \in [1, 4]$$

(i) f(x) is continuous in [1, 4]

(ii) f(x) is differentiable in (1, 4)

$$\therefore f'(x) = \frac{1}{2\sqrt{x^2 - x}} \times (2x - 1)$$

$f(4) = \sqrt{16 - 4} = \sqrt{12}$

$f(1) = \sqrt{1 - 1} = 0$

Therefore all the conditions of Lagrange's Mean Value theorem are satisfied.

$$\therefore f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$\Rightarrow \frac{2c - 1}{2\sqrt{c^2 - c}} = \frac{\sqrt{12} - 0}{4 - 1} = \frac{\sqrt{12}}{3}$$

Squaring both sides

$$\frac{(2c - 1)^2}{4(c^2 - c)} = \frac{\cancel{12}\ 4}{\cancel{9}\ 3}$$

$$\Rightarrow 3(4c^2 - 4c + 1) = 16(c^2 - c)$$

$$\Rightarrow 12c^2 - 12c + 3 = 16c^2 - 16c$$

$$\Rightarrow 4c^2 - 4c - 3 = 0$$

$$\Rightarrow 4c^2 - 6c + 2\ c - 3 = 0$$

$$\therefore (2c + 1)(2c - 3) = 0$$

$$c = -1/2 \ or \ c = 3/2$$

Clearly c = $3/2$ lies between 1 and 4.

$\therefore$ Lagrange's Mean Value theorem is verified.

(b) Let P(x, y) be a point on the conic, then

PS = e . PM i.e. $\sqrt{(x + 2)^2 + (y - 1)^2} = \frac{2}{\sqrt{3}}\left(\frac{2x - 3y + 1}{\sqrt{4 + 9}}\right)$

$$\Rightarrow 39(x^2 + y^2 + 4x - 2y + 5) = 4(4x^2 + 9y^2 + 1 - 12xy + 4x - 6y)$$

i.e. $23x^2 + 48xy + 3y^2 + 140x - 54y + 191 = 0$ is the required hyperbola.

Transverse axis passes through (–2, 1) and is perpendicular to Directrix,

2x – 3y + 1 = 0

TA: 3x + 2y + c = 0 where – 6 +2+c=0, c = 4; TA: 3x+2y+4=0

Question 5

(a) If $y = \left(\cot^{-1} x\right)^2$, *show that* $\left(1+x^2\right)^2 \frac{d^2y}{dx^2} + 2x\left(1+x^2\right)\frac{dy}{dx} = 2$ **[5]**

(b) Find the maximum volume of the cylinder which can be inscribed in a sphere of radius $3\sqrt{3}\,cm$. (Leave the answer in terms of ®) **[5]**

Comments of Examiners

(a) Some candidates took the derivative of $\cot^{-1}$ x as $\left(\frac{1}{1+x^2}\right)$ instead of -$\left(\frac{1}{1+x^2}\right)$ Use of the rule for composite function as well as chain rule was forgotten by several candidates.

(b) Volume of the cylinder had to be expressed in terms of a single variable (either r or h). Many candidates failed in this basic step. Also, second order derivative needs to be shown as negative for maximum volume. Some candidates were not able to do this.

Suggestions for teachers

- Derivatives of all forms of functions require continuous practice and review from time to time. Sufficient time needs to be spent on this topic.
- Students need to familiarise themselves with the area, perimeter, surface and volume of 2-Dimensional and 3-Dimensional figures in the syllabus. The function to be optimised needs to be expressed in terms of a single variable by using the given data. For maximum value *f′(x)=0* and *f″ (x) < 0*. This needs to be taught well.

MARKING SCHEME

Question 5.

(a) $y = \left(\cot^{-1} x\right)^2$

$$\therefore \frac{dy}{dx} = 2\cot^{-1} x \cdot \left(\frac{-1}{1+x^2}\right)$$

ie, $\left(1+x^2\right)\cdot\frac{dy}{dx} = -2\cot^{-1} x$

$$\Rightarrow \left(1+x^2\right)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = -2\left(\frac{-1}{1+x^2}\right)$$

ie, $\left(1+x^2\right)^2 \cdot \frac{d^2y}{dx^2} + 2x\left(1+x^2\right)\frac{dy}{dx} = 2$

(b) V = $\text{®} r^2 (2h)$

$= 2\text{®} h\left(27 - h^2\right)$

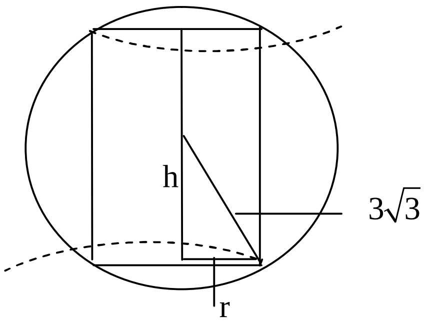
$= 54\text{®} h - 2\text{®} h^3$

$\frac{dv}{dh} = 54\text{®} - 6\text{®} h^2$

$\frac{dv}{dh} = 0 \Rightarrow h = 3$

Also, $\frac{d^2v}{dh^2} = -12\text{®} h\left(negative\right) : Hence\ \max .V$

$\therefore V_{\max} = 2\text{®}.3\left(27-9\right) = 108\text{®}$ cubic units.



Question 6

(a) Evaluate: $\int \frac{\cos^{-1} x}{x^2} dx$ **[5]**

(b) Find the area bounded by the curve $y = 2x - x^2$, and the line $y = x$. **[5]**

Comments of Examiners

(a) Some candidates attempted product rule of integration directly without any substitution and could not proceed beyond the first stage. Some took the $\int \frac{dx}{x\sqrt{1-x^2}}$ as $\sec^{-1} x$ which is incorrect.

A number of candidates who used an appropriate substitution did not give the final answer in terms of x but left it in terms of the new variable.

(b) Some candidates found $\int (y_1)\, dx$ or $\int (y_1 - y_2)\, dx$ within the wrong intervals (0, 2) where the curve cuts the x – axis.

Suggestions for teachers

- Integration by parts with all relevant substitutions and necessary changes needs good grounding and plenty of practice. If, cos Z = x and the integral reduces to $\int$SecZ.dz, then the answer should not be left in terms of Z as log $\left|\sec Z + \tan Z\right|$ + c but given in terms of x as log $\left|\frac{1}{x} + \frac{\sqrt{1-x^2}}{x}\right| + c$.

– Sketching of curves may not be necessary always but a rough sketch will always help the student to understand the requirements, the area required to be found, the points of intersection and the limits of the definite integral. Since the functions given are usually simple algebraic or trigonometric functions, the integration of such functions will not cause problems and the solution can be easily found.

MARKING SCHEME

Question 6.

(a) Let

$$I = \int \frac{\cos^{-1} x}{x^2} dx$$

Put $\cos^{-1} x = t$

$x = \cos t$

$dx = \sin t dt$

$$I = -\int \frac{t \sin t}{\cos^2 t} dt$$

$$= -\int t(\sec t \tan t) dt$$

$$= -\left[t \sec t - \int 1 \cdot \sec t \, dt \right]$$

$$= -t \sec t + \log |\sec t + \tan t| + c$$

$$= -\frac{1}{x}\cos^{-1} x + \log \left| \frac{1}{x} + \frac{\sqrt{1-x^2}}{x} \right| + c$$

$$= -\frac{1}{x}\cos^{-1} x + \log \left| \frac{1+\sqrt{1-x^2}}{x} \right| + c$$

(b) Solving, x = y, $y = 2x - x^2$,

$2x - x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

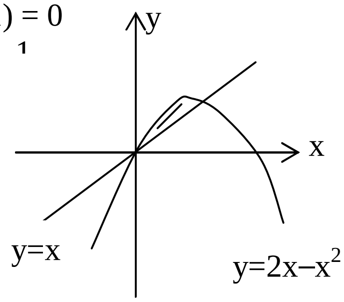
Required area $= \int_0^1 (y_1 - y_2)dx$

$= \int_0^1 \{(2x - x^2) - x\}dx$

$= \int_0^1 (x - x^2)dx$

$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= \left(\frac{1}{2} - \frac{1}{3}\right) - (0) = \frac{1}{6}$ sq. units



Question 7

(a) Find the Karl Pearson's co-efficient of correlation between *x* and *y* for the following data: **[5]**

x	16	18	21	20	22	26	27	15
y	22	25	24	26	25	30	33	14

(b) The following table shows the mean and standard deviation of the marks of Mathematics and Physics scored by the students in a school: **[5]**

	Mathematics	Physics
Mean	84	81
Standard Deviation	7	4

The correlation co-efficient between the given marks is 0·86. Estimate the likely marks in Physics if the marks in Mathematics are 92.

Comments of Examiners

(a) Since the means $\bar{x}$ and $\bar{y}$ are not whole numbers, use of the formula $\frac{\sum(dxdy)}{\sqrt{\sum dx^2}\sqrt{\sum dy^2}}$ by some candidates, taking approximate whole number values of the means was incorrect. Several candidates solved the problem using Spearman's method.

(b) Several candidates used incorrect formula. Some used the equation for the regression line of 'x on y' instead of 'y on x" to find y when x was given.

Suggestions for teachers

- Plenty of practice is required in order to solve problems on correlation.
- Teachers are to make sure the students understand the need for the appropriate formula at appropriate situations.

MARKING SCHEME

Question 7.

(a)

x	y	dx=x-20	dy=y-25	dx^2	dy^2	dx×dy
16	22	–4	–3	16	9	12
18	25	–2	0	4	0	0
21	24	1	–1	1	1	–1
20	26	0	1	0	1	0
22	25	2	0	4	0	0
26	30	6	5	36	25	30
27	33	7	8	49	64	56
15	14	–5	–11	25	121	55
		5	–1	135	221	152

$$\therefore r = \frac{n\sum(dx \cdot dy) - \sum dx \times \sum dy}{\sqrt{n\sum dx^2 - \left(\sum dx\right)^2}\sqrt{n\sum dy^2 - \left(\sum dy\right)^2}}$$

$$= \frac{8\times152-(5\times-1)}{\sqrt{8\times135-5^2}\times\sqrt{8\times221-(-1)^2}}$$

$$= \frac{1216+5}{\sqrt{1080-25}\sqrt{1768-1}} = 0.894$$

(b) Let Mean marks in mathematics $(\bar{x}) = 84$

Mean marks in Physics $(\bar{y}) = 81$

$\pm_x = 7$, $\pm_y = 4$

r = 0.86

$b_{yx} = r \times \frac{\pm_y}{\pm_x}$

$= 0{\cdot}86 \times \frac{4}{7} = 0 \cdot 49$

or 86/175

$\therefore$ Regression line of y on x be $y - \bar{y} = b_{yx}(x - \bar{x})$

y – 81 = 0.49(x–84)

y = 0.49x – 41.16 + 81

y = 0.49x + 39.84

whereas x = 92, y = 0.49 ×92 + 39.84

y = 84.92

Question 8

(a) Bag A contains three red and four white balls; bag B contains two red and three white balls. If one ball is drawn from bag A and two balls from bag B, find the probability that: **[5]**

(i) One ball is red and two balls are white;

(ii) All the three balls are of the same colour.

(b) Three persons, Aman, Bipin and Mohan attempt a Mathematics problem independently. The odds in favour of Aman and Mohan solving the problem are 3:2 and 4:1 respectively and the odds against Bipin solving the problem are 2:1. Find: **[5]**

(i) The probability that all the three will solve the problem.

(ii) The probability that problem will be solved.

Comments of Examiners

(a)(i) & (ii) The product and summation laws of probability and their application was not clear to many candidates. Both parts (i) and (ii) required the application of these laws jointly.
Students failed to identify or exhaust all possible cases.
(b)(i) The concept of odds in favour or against was not clear to several candidates. Some candidates took the probability of all three solving as $P_1+P_2+P_3$ instead of $P_1 x P_2 x P_3$.
(ii) Some candidates failed to realise that 'the probability that the problem will be solved', means, the probability that at least one solves the problem which is, 1–P.

Suggestions for teachers

- Explain the probability laws correctly. Students need to understand the problem and identify the cases that satisfy the situation.
- Students need to know how to obtain probabilities from odds stated, e.g. if the odds one m:n in favour, then probability of occurrence is $\frac{m}{m+n}$. The rest of the problem follows the usual combination of sum and product laws.

MARKING SCHEME

Question 8.

(a) (i) P(1 red ball from A and 2 white from B)= $\frac{3C_1}{7C_1}\cdot\frac{3C_2}{5C_2}=\frac{9}{70}$

P(1W from A and R & W from B) = $\frac{4C_1}{7C_1}\cdot\frac{2C_1\cdot 3C_1}{5C_2}=\frac{24}{70}$

$$\therefore P(one\,\mathrm{Re}d\ \&\ twoWhite)=\frac{9}{70}+\frac{24}{70}=\frac{33}{70}$$

(ii) P(all same colour) = P(all three red)+P(all three white)

$$=\frac{3C_1}{7C_1}\cdot\frac{2C_2}{5C_2}+\frac{4C_1}{7C_1}\cdot\frac{3C_2}{5C_2}$$

$$=\frac{3}{7}\cdot\frac{1}{10}+\frac{4}{7}\cdot\frac{3}{10}$$

$$=\frac{15}{70}\,or\,\frac{3}{14}\text{ or }0.21$$

(b) $P(E_1)=\frac{3}{5}, P(E_2)=\frac{1}{3}, P(E_3)=\frac{4}{5}$

(i) $P(E_1\cap E_2\cap E_3)=P(E_1)\cdot P(E_2)\cdot P(E_3)$

$$=\frac{3}{5}\cdot\frac{1}{3}\cdot\frac{4}{5}=\frac{4}{25}$$

(ii) P(at least one solves) = 1 – P$\left(\overline{E_1} \cap \overline{E_2} \cap \overline{E_3}\right)$

$$= 1 - \frac{2}{5}\cdot\frac{2}{3}\cdot\frac{1}{5} = 1 - \frac{4}{75} = \frac{71}{75}$$

Question 9

(a) Find the locus of the complex number z = $x + iy$, satisfying relations arg(z – 1) = ®⁄4 **[5]**

and $|z-2-3i| = 2$. Illustrate the locus on the Argand plane.

(b) Solve the following differential equation: **[5]**

$ye^y\,dx = (y^3+2xe^y)\,dy$, given that $x = 0$, $y = 1$.

Comments of Examiners

(a) Sketching of loci was inaccurate in several cases. Some candidates were confused about locus satisfying both conditions and failed to find the points of intersection.
Sketching was done on separate axes by several candidates, instead of the same axes and graph.
(b) A number of candidates could not write the given equation in the standard form: $\frac{dx}{dy} + P\cdot x = Q$ (where P, Q are functions in y). Some, who managed, took 'P' as $\frac{2}{y}$ instead of $-\frac{2}{y}$. This resulted in incorrect Integrating Factor.

Suggestions for teachers

- Sketching of straight lines and curves (circle, conics etc.) should be practiced regularly.
- Post differentiation and integration solving of differential equations needs a lot of practice.

MARKING SCHEME

Question 9.

(a) Let z = x + iy

arg(z-1) = $\frac{®}{4}$

arg(x +iy–1)= $\frac{®}{4}$

$\tan^{-1}\frac{y}{x-1} = \frac{®}{4}$

$\frac{y}{x-1} = 1 \quad \therefore y = x - 1 ----(i)$

$|z-2-3i| = 2$

$\Rightarrow |x + iy - 2 - 3i| = 2$

$(x-2)^2 + (y-3)^2 = 4$

$(x-2)^2 + (x-1-3)^2 = 4 \;\; (\mathit{from}\,(i)\;\; y = x-1)$

$x^2 - 4x + 4 + x^2 - 8x + 16 = 4$

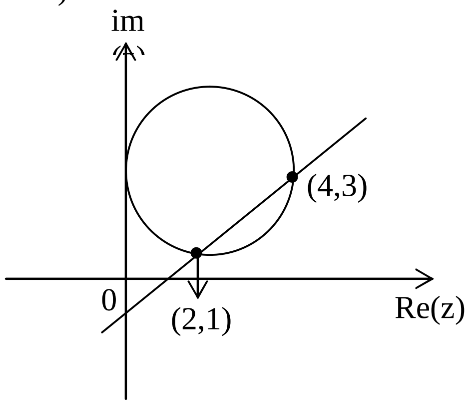
$2x^2 - 12x + 16 = 0$

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$

$x = 4$ or 2

$y = 3$ or 1



$\therefore$ the locus will be points (2, 1) and (4, 3)

$\therefore x = [y^2(e]^{-1} - e^{-y})$

SECTION B

Question 10

(a) If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them, then show that $|\vec{a} - \vec{b}| = 2\sin\frac{\theta}{2}$. **[5]**

(b) Find the value of λ for which the four points A, B, C, D with position vectors $-\hat{j} - \hat{k}$; $4\hat{i} + 5\hat{j} + \lambda\hat{k}$; $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$ are coplanar. **[5]**

<u>Comments of Examiners</u>

(a) Many candidates who attempted this part, failed to convincingly prove the rider. Those who erred did so mainly in converting 1 - cosθ, into $2\text{Sin}^2\left(\frac{-}{2}\right)$. Vector notations were also not used by several candidates.

(b) Some candidates erred in finding vectors $\overrightarrow{AB} \cdot \overrightarrow{AC}$, or $\overrightarrow{AD}$. Scalar triple product as a determinant was incorrectly expanded by some candidates.

<u>Suggestions for teachers</u>

- Vector notations, usage, dot and cross products in terms of the vectors or their components need to be taught well and in detail. Basic rules of trigonometry need to be revised.
- The students must understand that $\overrightarrow{AB}$ = Position vector of B – Position vector of A.

- The fact that the scalar Triple Product of three co-initial vectors gives the volume of the parallelopiped with these vectors as the edges, needs explanation. If this Determinant (i.e. Volume) is zero, the conclusion that the vectors are coplanar is evident.

MARKING SCHEME

Question 10.

(a) $\left|\vec{a}-\vec{b}\right|^2 = |\vec{a}|^2 - 2|\vec{a}|\cdot|\vec{b}|\cos\theta + |\vec{b}|^2$

$\vec{a}, \vec{b}$ are unit vectors $\Rightarrow |\vec{a}| = |\vec{b}| = 1$

$$\left|\vec{a}-\vec{b}\right|^2 = 1 - 2\cos\theta + 1$$
$$= 2 - 2\cos\theta$$
$$= 2(1-\cos\theta)$$
$$= 2 \times 2\sin^2\frac{\theta}{2}$$
$$\left|\vec{a}-\vec{b}\right| = 2\sin\left(\theta/2\right)$$

(b) $\overrightarrow{AB} = (4i+5j+\lambda k) - (-j-k) = 4i+6j+(\lambda+1)k$

$\overrightarrow{AC} = (3i+9j+4k) - (-j-k) = 3i+10j+5k$

$\overrightarrow{AD} = (-4i+4j+4k) - (-j-k) = -4i+5j+5k$

$\because$ Vectors are coplanar, $\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right] = 0$

ie. $\begin{vmatrix} 4 & 6 & \lambda+1 \\ 3 & 10 & 5 \\ -4 & 5 & 5 \end{vmatrix} = 0$

ie $4(50-25) - 6(15+20) + (\lambda+1)(15+40) = 0$

$55(\lambda+1) = 110 \quad \therefore \lambda = 1$

Question 11

(a) Find the equation of a line passing through the point (−1, 3, −2) and perpendicular to the lines: $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$. **[5]**

(b) Find the equations of planes parallel to the plane $2x - 4y + 4z = 7$ and which are at a distance of five units from the point (3, −1, 2). **[5]**

Comments of Examiners

(a) Candidates who attempted by vector method got confused between equation of a line and equation of a plane in vector form. Some candidates did not use the cross-product of perpendicular vectors correctly.
(b) The distance formula of a point from a plane did not include the modulus symbol in several cases. Hence only one value of 'λ' was obtained, whereas two were required (i.e. two planes).
Some candidates also concluded that if $(18+\lambda)^2=900$, then $18+\lambda = 30$, whereas it should be ± 30.

Suggestions for teachers

- Use of Cartesian and vector methods for solving must be separately explained and compared for complete understanding.
- Distance of point from point, point from line, point from plane as well as the shortest distance between two lines are topics that need careful study and understanding.

MARKING SCHEME

Question 11.

(a) The direction vector of the required line is cross product of the direction vectors of the given lines.

$$\therefore \vec{p} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -3 & 2 & 5 \end{vmatrix}$$

$$= \hat{i}(10-6) - \hat{j}(5+9) + \hat{k}(2+6)$$

$$= 4\hat{i} - 14\hat{j} + 8\hat{k}$$

$$\therefore \text{equation of the required line is } \frac{x+1}{4}=\frac{y-3}{-14}=\frac{z+2}{8}$$

$$\Rightarrow \frac{x+1}{2}=\frac{y-3}{-7}=\frac{z+2}{4}$$

(b)	Equation of the plane parallel to the given plane is 2x–4y+4z+λ =0. This plane is at the distance of five units from the point (3, –1, 2) if $\left\lvert \frac{6+4+8+\lambda}{\sqrt{4+16+16}} \right\rvert = 5$ $\frac{\lvert 18+\lambda \rvert}{\sqrt{36}} = 5$ $(18+\lambda)^2 = 25\times 36$ $18+\lambda = \pm 30$ $\lambda = 12, -48$ Equations of the required plane are: 2x – 4y + 4z + 12 = 0 and 2x – 4y + 4z – 48 = 0 or x–2y + 2z + 6 = 0 and x – 2y + 2z – 24 = 0

Question 12

(a) If the sum and the product of the mean and variance of a Binomial Distribution are 1·8 and 0·8 respectively, find the probability distribution and the probability of at least one success. **[5]**

(b) For A, B and C, the chances of being selected as the manager of a firm are 4 : 1 : 2, respectively. The probabilities for them to introduce a radical change in the marketing strategy are 0·3, 0·8 and 0·5 respectively. If a change takes place; find the probability that it is due to the appointment of B. **[5]**

Comments of Examiners

(a) From the given, np + npq = 1.8 and np.(npq) = 0.8, some candidates could not solve for n, p, q.
The probability distribution was not correctly expressed by many candidates. Some candidates did not know how to find the probability of at least one successful outcome.
(b) Many candidates used Baye's theorem correctly but took the probabilities of A, B, C, as 4, 1, 2, instead of fractions $\frac{4}{7}, \frac{1}{7}, \frac{2}{7}$.
Some candidates did not implement the theorem correctly.

Suggestions for teachers

- Students must be made to revise standard deviation or variance. While solving for n, p, q it must be noted that p+q=1. Practice in solving equations in two or three unknowns is a must.
- Probabilities are ratios and not numbers – this must be made clear. P(E/A) = P(E). P(A/E) must be explained as well as the implementation of Baye's theorem.

MARKING SCHEME

Question 12.

(a) Mean = np variance = npq

np + npq = 1.8 …………………. (i)

np (npq) = 0.8 …………………. (ii)

Solving, $q = \frac{4}{5} \quad \therefore p = \frac{1}{5}, n = 5$

$\therefore$ Distributionis $:(q+p)^n$ ie. $\left(\frac{4}{5}+\frac{1}{5}\right)^5$

P(at least one success) = $1-\left(\frac{4}{5}\right)^5 = 1-\cdot 33$

= 0.67 or 2101/3125

(b) Let events be

E_1 = A is selected as manager

E_2 = B is selected as manager

E_3 = C is selected as manager

E = radical change in the marketing strategy

$P(E_1) = 4/7,\ P(E_2) = \frac{1}{7},\ P(E_3) = 2/7$

By Baye's theorem

$$P(E_2/E) = \frac{P(E_2)P(E/E_2)}{(P(E_1)P(E/E_1) + P(E_2)P(E/E_2) + P(E_3)P(E/E_3))}$$

as $P\left(\frac{E}{E_1}\right) = 0.3,\ P\left(\frac{E}{E_2}\right) = 0\cdot 8$ and $P\left(\frac{E}{E_3}\right) = 0\cdot 5$

Applying Baye's theorem

$$P(E_2/E) = \frac{(1/7\times 0\cdot 8)}{(4/7\times 0\cdot 3 + 1/7\times 0\cdot 8 + 2/7\times 0\cdot 5)}$$

$$P(E_2/E) = \frac{4}{15}$$

SECTION C

Question 13

(a) If Mr. Nirav deposits ₹250 at the beginning of each month in an account that pays an interest of 6% per annum compounded monthly, how many months will be required for the deposit to amount to at least ₹6,390 ? **[5]**

(b) A mill owner buys two types of machines A and B for his mill. Machine A occupies 1000 sqm of area and requires 12 men to operate it; while machine B occupies 1200 sqm of area and requires 8 men to operate it. The owner has 7600 sqm of area available and 72 men to operate the machines. If machine A produces 50 units and machine B produces 40 units daily, how many machines of each type should he buy to maximise the daily output? Use Linear Programming to find the solution. **[5]**

Comments of Examiners

(a) Some candidates took Amount for Immediate Annuity formula $\frac{A}{i}\left\{(1+i)^n - 1\right\}$ instead of Annuity Due formula: $\frac{A}{i}(1+i)\left\{(1+i)^n - 1\right\}$. Some substitutions were also not correct.

(b) In some cases, all the constraints were not stated or used and hence coordinates of all feasible points were not obtained.

Suggestions for teachers

- Help students to distinguish between Amount of Annuity, Present Value of Annuity, Immediate Annuity and Annuity Due. Also, the reasons for the different formulae must be explained.
- The optimum function and all possible constraints in the form of inequations have to be put down from what is stated in the problem. Students must be made to solve the constraints equations in pairs to obtain all feasible points leading to maximum or minimum value of desired function.

MARKING SCHEME

Question 13.

(a) $a = 250, \quad i = \frac{6}{12 \times 100} = 0.005 \text{ and } n = 12m$ months

$$S = \frac{a}{i}(1+i)\{(1+i)^n - 1\}$$

$$6390 = \frac{250}{0.005}(1.005)\{(1.005)^{12m} - 1\}$$

$$6390 = \frac{250}{0.005}(1.005)\{(1.005)^{12m} - 1\}$$

$$0.1271 = \{(1.005)^{12m} - 1\}$$

$$1.1271 = (1.005)^{12m}$$

$$\log_e 1.1271 = \log_e (1.005)^{12m}$$

$$\frac{\log_e 1.1271}{\log_e (1.005)} = 12m$$

$$12m = 23.98 \sim 24$$

23.98 or 24 months

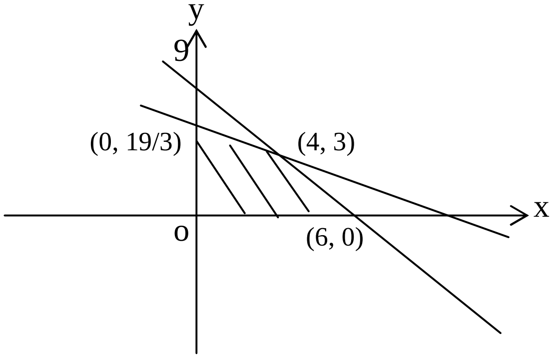
(b) Let there be x machines of type A and y machines of type B. Daily output, Z = 50x + 40y.

Constraints are:

$1000x + 1200y \leq 7600$

$12x + 8y \leq 72$

$x \geq 0, y \geq 0$



Feasible points are (6, 0), (4, 3), (0, 19/3)

At (6, 0), Z = 300

At (4, 3), Z = 320

$\therefore$ for maximum output there should be 4 machines and 3 machines respectively of type A and B.

Question 14

(a) A bill of ₹60,000 was drawn on 1st April 2011 at 4 months and discounted for ₹58,560 at a bank. If the rate of interest was 12% per annum, on what date was the bill discounted? **[5]**

(b) A company produces a commodity with ₹24,000 fixed cost. The variable cost is estimated to be 25% of the total revenue recovered on selling the product at a rate of ₹8 per unit. Find the following: **[5]**

(i) Cost function

(ii) Revenue function

(iii) Breakeven point.

Comments of Examiners

(a) Some candidates omitted to include the Days of Grace and so got the wrong value of 'n'. Some candidates used True Discount formula instead of Banker's Discount or relevant formula. Several candidates had no idea how to get the Date of Discounting.
(b) (i) In some cases, variable cost was incorrectly calculated and hence the total cost.
(ii) Some candidates erred in obtaining Revenue function.
(iii) The condition for Breakeven point was forgotten by several candidates.

Suggestions for teachers

- All relevant terms and formulae need to be taught well for complete understanding. Legal due date includes three days of grace.
- While using any formula using n, it should be made clear that it does not stand for the entire period but only for the unexpired period from Date of Discounting to Legal Due Date.

MARKING SCHEME

Question 14.

(a) Banker's Discount, B.D = 60,000 – 58560 = Rs 1440

Let t be unexpired period in years

B.D = Ani

$$1440 = \frac{60000(12)t}{100} \Rightarrow t = \frac{1}{5} \text{ year or } \frac{1}{5} \times 365 = 73 \text{ days}$$

Legal due date of bill

1st April 2011 + 4 months +3 days of grace = 4th August 2011

The bill was cashed 73 days before 4th August.

04 days in August

31 days in July

30 days in June

08 days in May which comes to 23rd May 2011 as the date.

(b) Let x be the number of units produced and sold.

Price per unit = Rs. 8

Revenue function =R (x)= 8x

Variable cost of the x units = 25% of 8x

= Rs. 2x

Total cost =c (x)= fixed cost + variable cost

Total cost= c (x) =24000 + 2x At break even point : R(x) = c(x)

∴ 8x =24000+2x

x= 4000…

Question 15

(a) The price index for the following data for the year 2011 taking 2001 as the base year was 127. The simple average of price relatives method was used. Find the value of *x*: **[5]**

Items	A	B	C	D	E	F
Price (₹ per unit) in year 2001	80	70	50	20	18	25
Price (₹per unit) in year 2011	100	87·50	61	22	*x*	32·50

(b) The profits of a paper bag manufacturing company (in lakhs of rupees) during each month of a year are: **[5]**

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
Profit	1·2	0·8	1·4	1·6	2·0	2·4	3·6	4·8	3·4	1·8	0·8	1·2

Plot the given data on a graph sheet. Calculate the four monthly moving averages and plot these on the same graph sheet.

Comments of Examiners

(a) Errors were committed by several candidates as the instructions were not followed. Instead of using Price-relatives, some candidates used simple aggregate of prices method.

(b) Moving averages were mostly correctly calculated but for plotting, centred moving averages were required. Some candidates did not use the centred averages. In some cases, the graphs were not neat.

Suggestions for teachers

– Moving Averages need to be calculated correct to two decimal places. For plotting, centred averages correct to one decimal place is sufficient. The axes should be labelled, plotting and sketching should be as neat as possible and the graph should be given a caption.

MARKING SCHEME

Question 15.

(a) Using simple average of price relatives, the price index for 2011 taking 2001 as base year, was 127 from the following data find the value of x:

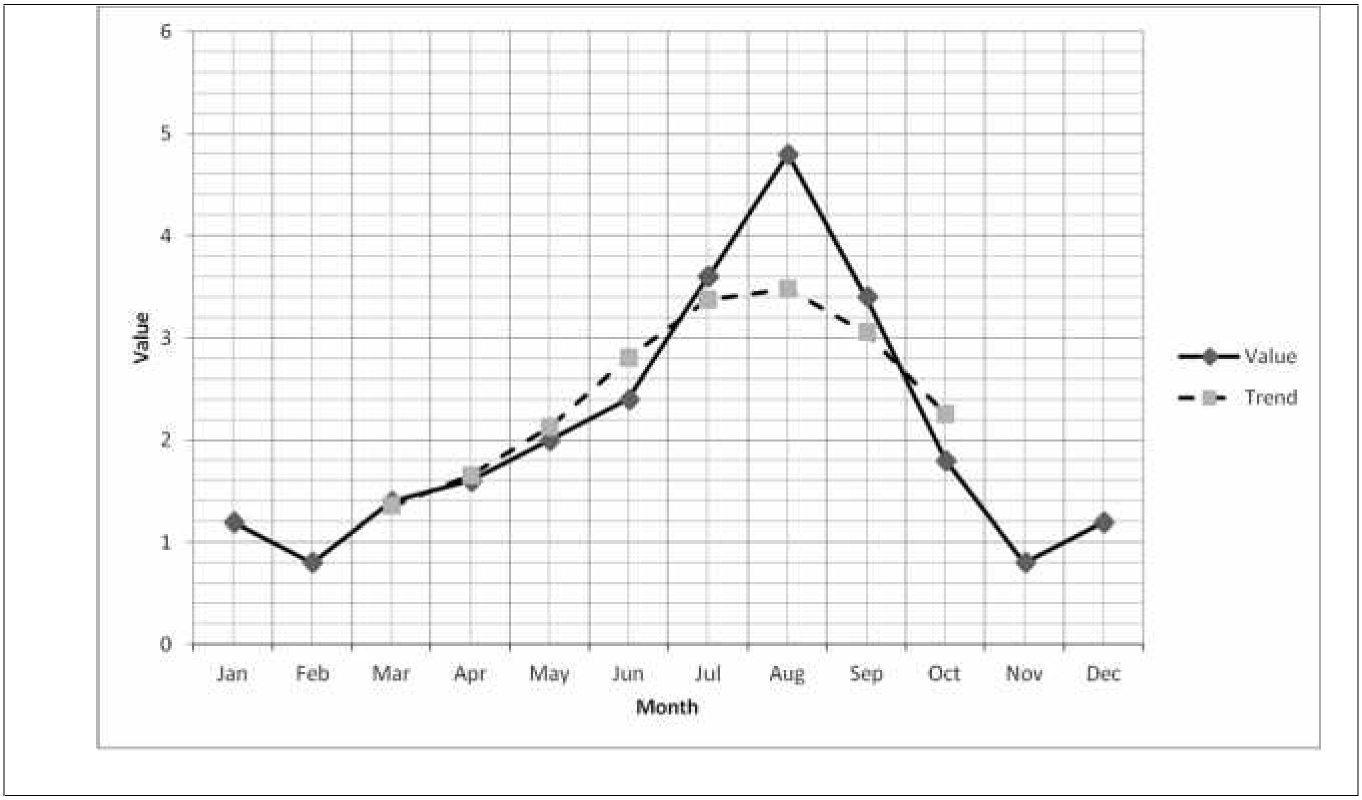
P_0	80	70	50	20	18	25
P_1	100	87.50	61	22	X	32.50
PR	125	125	122	110	$\frac{100x}{18}$	130

$$\frac{\sum PR}{N} = 127$$

i.e. $612 + \frac{100x}{18} = 127 \times 6 \quad \therefore \quad x = 27$

(b)

Months	profit	4 monthly total	4 monthly average	4 monthly centred moving average
JAN	1.2	-	-	-
FEB	0.8	-	-	-
		5	1.25	
MAR	1.4			1.35
		5.8	1.45	
APR	1.6			1.65
		7.4	1.85	
MAY	2.0			2.125
		9.6	2.4	
JUN	2.4			2.8
		12.8	3.2	
JULY	3.6			3.375
		14.2	3.55	
AUG	4.8			3.475
		13.6	3.4	
SEP	3.4			3.05
		10.8	2.7	
OCT	1.8			2.25
		7.2	1.8	
NOV	0.8			-
		-	-	
DEC	1.2			-



GENERAL COMMENTS:

(a) Topics found difficult by candidates in the Question Paper:

- Indefinite Integrals (use of substitution or integration by parts)
- Definite Integrals – use of properties.
- Inverse Circular Functions (formulae and relations)
- Differential Equations (solving Linear Differential Equations)
- Vectors – in general
- Annuity (use of appropriate formula)
- Conics in general (Hyperbola in particular)
- Probability – use of sum and product laws and identifying all cases.
- Maxima and Minima (expressing volume of cylinder in terms of r or h only)

(b) Concepts between which candidates got confused:

- For $\int_o^a f(x)\,dx$, obtaining f(a-x) from f(x).
- Regression lines: y on x and x on y
- Sum and product laws of probability
- 3 – D: parallel planes and perpendicular forms.
- Conditional probability property in Baye's theorem.
- Price Index by aggregate and Price Relative methods.
- Differences between and usage of formulae for BD, TD, BG, DV etc.

(c) Suggestions for students:

- Study the entire syllabus thoroughly and revise from time to time. Concepts of Class XI must be revised and integrated with the Class XII syllabus.
- Develop logical and reasoning skills to have a clear understanding.
- Revise all topics and formulae involved and make a chapter wise or topic-wise list of these.
- Make wise choices from the options available in the question paper.
- Be methodical and neat in your working.